

# Characteristic Impedance and Effective Permittivity of Modified Microstrip Line for High Power Transmission

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**Abstract** — In a quasi-TEM approximation, the calculation of the characteristic impedance and effective permittivity of a modified microstrip line with the upper strip conductor edges turned up from the substrate is presented. Both these theoretically derived quantities are compared with experimental values. To further facilitate the analysis and synthesis of the modified microstrip line, closed-form approximations to the characteristic impedance and effective permittivity have been found containing elementary functions only. The modified microstrip can be used in the construction of lines capable of transmitting high power on substrates having high loss factor and low thermal conductivity, for example, on low-cost plastics.

## I. INTRODUCTION

MICROSTRIP LINES are mostly used in microwave circuits and systems processing signals at low power levels. Usually, the power transmitted by this line is of the order of tens of milliwatts, and rarely does it reach units of watts. The average power transmitted over the microstrip line is limited by the rise of temperature caused by losses in conductors and in the dielectric substrate, while the peak power is limited by the peak voltage causing dielectric breakdown. One review and discussion of the effects limiting the transmitted power in the microstrip line can be found in [1].

In special cases, power of the order of tens or hundreds of watts may be required to be transmitted. This requirement occurred in a power transistor amplifier built on a laminated plastic substrate when a power of 150 W with total longitudinal current of 24 A had to be transmitted [2]. The conductors of the line must have a high current-carrying capability, and the temperature of the line must not rise too high; otherwise, the substrate would be destroyed. The highest current density in the upper strip conductor is at its edges, as is the highest electric field intensity, resulting in a corresponding rise of the substrate temperature [3], [4]. The unfavorable effects quoted above can be minimized by choosing a suitable cross section of the microstrip line, as shown in Fig. 1 [2]. It differs from the standard microstrip line by the edges of the upper strip conductor being turned up from the dielectric substrate surface.

In this article, a calculation by a quasi-TEM approximation of the characteristic impedance and effective permittivity of this modified microstrip line suitable for high power transmission is presented and compared with experimental values. From the satisfactory agreement, we conclude that the derived results are appropriate solutions to the problem. To further facilitate analysis and synthesis of the modified microstrip line, closed-form approximations have been found which contain elementary functions only. Having in mind the range of values of the characteristic impedance met most often in practice, the ranges of values of  $\epsilon_r$ ,  $w/h$ , and  $r/h$  are chosen such that they include "wide" microstrip lines. In addition, the approximations mentioned above are also suitable for calculation of the attenuation constant resulting from dielectric losses, or of the corresponding quality factor.

## II. CALCULATION OF THE CHARACTERISTIC IMPEDANCE AND EFFECTIVE PERMITTIVITY

We shall calculate the characteristic impedance and effective permittivity of the modified microstrip line with cross section of Fig. 1 by a quasi-TEM approximation for a conductor thickness negligible in comparison with the other dimensions, i.e.,  $t \approx 0$ . In practice, the so-called wide microstrip lines of this type are important for values  $2.5 \leq w/h \leq 7.0$ ,  $0.3 \leq r/h \leq 2.0$ , and  $1 \leq \epsilon_r \leq 10$ . With regard to the ranges of the particular variables, the characteristic impedance  $Z$  and the effective permittivity  $\epsilon_e$  are frequency independent up to about 3 GHz.

Fig. 2 represents the equipotential lines and lines of force of the electric field in one half of the modified microstrip line cross section with  $\epsilon_r = 1$  measured on a model using an electroconductive foil. It is obvious that the electric field can be considered, without great error, as one composed of the three separated fields shown in Fig. 3. However, the modified microstrip line  $\epsilon_r > 1$  results in concentration of the electric field in the dielectric in the first and second regions, and deforms the lines of force in the vicinity of the air-dielectric boundary. In the first region, the field is represented by the field of an ideal capacitor with plane electrodes and homogeneous dielectric of permittivity  $\epsilon_r$ . In the second region, the field is represented by half of the field of the microstrip line with circular upper conductor [5], the lines of force radiating

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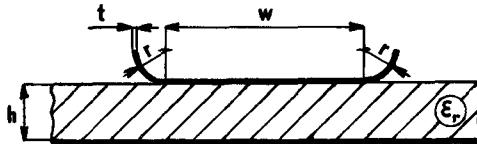
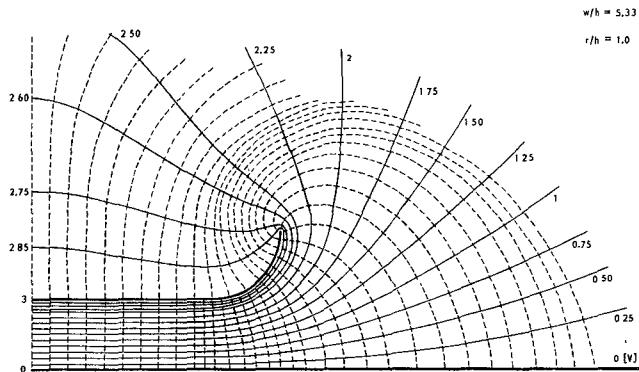


Fig. 1. Cross section of the modified microstrip line.

Fig. 2. Arrangement of the equipotential lines and lines of force of the TEM wave when  $w/h = 5.33$ ,  $r/h = 1$ ,  $\epsilon_r = 1$ .

from the upper side of the upper strip conductor (Fig. 3) having a form similar to those radiating from the circular conductor surface. The field in the third region is represented by lines of force of the microstrip line which radiate from the upper side of the upper strip conductor having width  $(w + \pi r)$ , where  $0 \leq u \leq w/2$ .

Let  $Z_i$  be the partial characteristic impedance of the  $i$ th region ( $i = 1, 2, 3$ ), now including also the second half of the line cross section symmetrically placed with respect to the axis  $v$  in Fig. 3. Then, the characteristic impedance of the modified microstrip line is

$$Z = \frac{Z_1 Z_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \quad (1)$$

and its effective permittivity is

$$\epsilon_e = \left( \frac{Z_0}{Z} \right)^2 \quad (2)$$

where  $Z_0$  is  $Z$  if  $\epsilon_r = 1$ .

The characteristic impedance of the first region is

$$Z_1 = \frac{120\pi}{\sqrt{\epsilon_r}} \cdot \frac{h}{w}. \quad (3)$$

The characteristic impedance of the second region is equal to the characteristic impedance of the microstrip line with a circular upper conductor having radius  $r$ . The formula for this impedance can be found in [5] ("round microstrip"). Accordingly,

$$Z_2 = 59.952 \sqrt{L \left[ L + \sum_{n=0}^{\infty} \left( \frac{1-\epsilon_r}{1+\epsilon_r} \right)^{n+1} \ln \left( 1 + \frac{2}{n+s} \right) \right]} \quad (4)$$

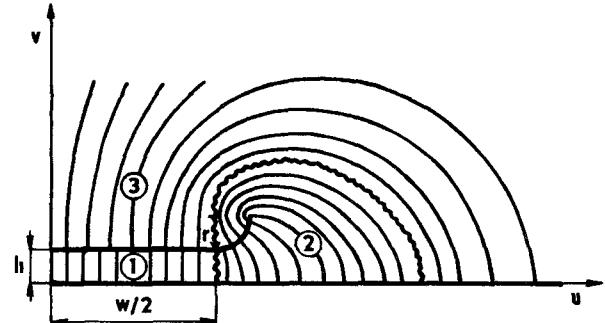


Fig. 3. Substitution of the three separated fields for the electric field of the TEM wave in Fig. 2.

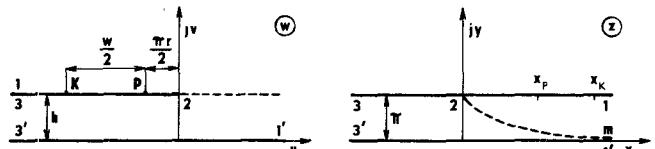


Fig. 4. Conformal transformation of a plate capacitor with lower electrode extending to infinity in both directions and the upper one extending to infinity in one direction only.

where

$$L = \ln \left[ 1 + \frac{1}{2s} (1 + \sqrt{1+4s}) \right] \quad (5)$$

$$s = \frac{r}{2h}. \quad (6)$$

When  $\epsilon_r$  and  $r/h$  are within intervals quoted above, it is sufficient in the alternating series in (4) to take  $n = 50$ . In this case, the calculated difference between the total sum and the sum of the first  $n$  members of the series is less than 0.001 percent.

To calculate  $Z_3$  and taking into consideration that  $3.4 \leq (w + \pi r)/h \leq 13.3$ , we can use the model of a parallel-plate capacitor shown in Fig. 4. By the Schwarz-Christoffel and the following logarithmic transformation [6], the polygon 1-2-3 from the  $w$  plane is transferred into two parallel lines in the  $z$  plane:

$$w = \frac{h}{\pi} (e^z + z + 1). \quad (7)$$

The straight line  $v = h$  for  $u \geq 0$  from the  $w$  plane transforms to the curve  $m$  in the  $z$  plane described by

$$e^x = \frac{\pi - y}{\sin y}. \quad (8)$$

Two points  $P$  and  $K$  lying on the upper side of the upper strip conductor in the  $w$  plane are transformed into the  $z$  plane, and their  $x$  coordinates  $x_p$  and  $x_k$  are given by solution of the following equation:

$$e^x - x = 1 + \frac{\pi}{h} |u|. \quad (9)$$

For  $x_p$  and  $x_k$  sufficiently large, the lines of force of the electric field radiating from the upper conductor within the range  $(x_p, x_k)$  in the  $z$  plane will intersect the curve  $m$ ,

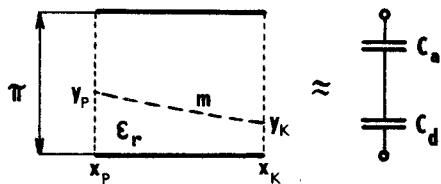


Fig. 5. Approximate  $z$  plane equivalent of the third region of the  $w$  plane.

which represents the surface of the dielectric in the microstrip line, almost perpendicularly. The capacitance corresponding to this field in the microstrip line can be simply determined as the capacitance of two serially connected capacitors in the  $z$  plane (Fig. 5). Their electrodes are  $(x_K - x_p)$  wide and 1 m long. The dielectric of the first capacitor is air and the dielectric of the second is the substrate. In both capacitors, the spacing between their electrodes varies in the  $x$  direction. Keeping in mind the chosen range of  $w/h$  and  $r/h$ , the slope of the line  $m$  if  $x$  is within  $(x_p, x_K)$  can be considered approximately constant, and the equation of the line can be written as

$$y = px + q \quad (10)$$

where

$$p = \frac{y_K - y_p}{x_K - x_p} \quad (11)$$

$$q = y_p - px_p \quad (12)$$

$$y_p = \frac{\pi}{2 + x_p + \frac{\pi^2 r}{2h}} \quad (13)$$

$$y_K = \frac{\pi}{2 + x_K + \frac{\pi^2 r}{2h} + \frac{\pi w}{2h}}. \quad (14)$$

In (8), we have used a simplification  $\sin y \approx y$  as  $y \leq 0.4$ . The capacitance of the first capacitor is

$$C_a = \epsilon_0 \int_{x_p}^{x_K} \frac{dx}{\pi - px - q} = \frac{\epsilon_0}{p} \ln \left| \frac{px_p + q - \pi}{px_K + q - \pi} \right| \quad (15)$$

and that of the second one is

$$C_d = \epsilon_0 \epsilon_r \int_{x_p}^{x_K} \frac{dx}{px + q} = \frac{\epsilon_0 \epsilon_r}{p} \ln \left| \frac{px_K + q}{px_p + q} \right| = \epsilon_r C_{d0}. \quad (16)$$

The capacitance corresponding to the third region in Fig. 3 supplemented by the dielectric substrate is now

$$C = \frac{C_a C_d}{C_a + C_d}. \quad (17)$$

Thus

$$Z_3 = \frac{1}{2c \sqrt{C C_0}} \quad (18)$$

where  $C_0$  is  $C$  if  $\epsilon_r = 1$ , i.e.,

$$C_0 = \frac{C_a C_{d0}}{C_a + C_{d0}}. \quad (19)$$

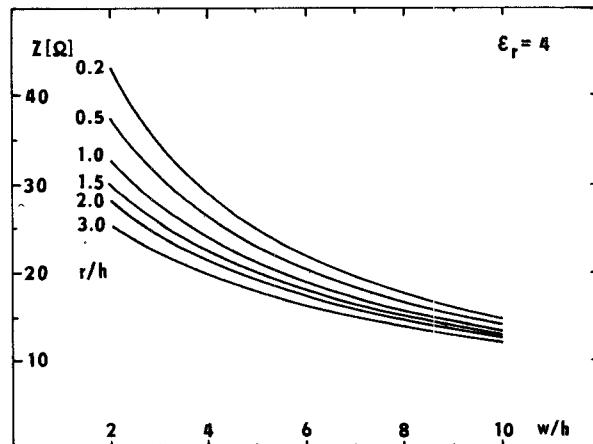


Fig. 6. Characteristic impedance of the modified microstrip line.

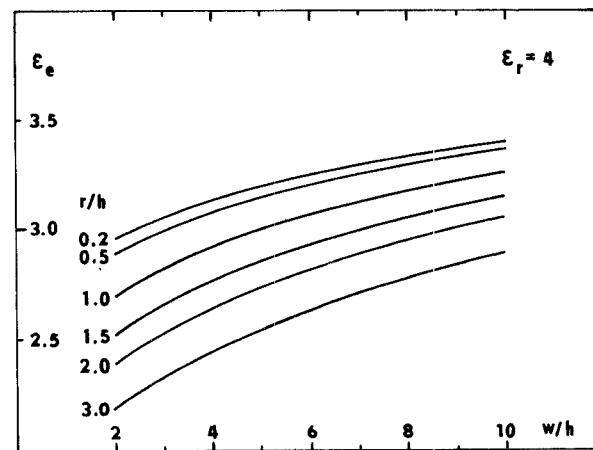


Fig. 7. Effective permittivity of the modified microstrip line.

$Z_1$  and  $Z_2$  contribute substantially to the characteristic impedance of the modified microstrip line. The contribution of the partial characteristic impedance  $Z_3$  is small and does not exceed 5 percent of the total  $Z$ .

### III. NUMERICAL AND EXPERIMENTAL RESULTS

The curves of characteristic impedance and effective permittivity of the modified microstrip line with  $\epsilon_r = 4$  are plotted in Figs. 6 and 7. In addition to  $w/h$ , both quantities can be affected by  $r/h$ . The characteristic impedance of the modified microstrip line is found to lie between two values—the characteristic impedances of the microstrip line having the width of the upper strip conductor  $w$  and that with width  $(w + \pi r)$  calculated according to [7]. Turning up the edges of the upper strip conductor from the dielectric substrate surface causes the characteristic impedance to increase and the effective permittivity to decrease. This arrangement of the line increases the number of degrees of freedom by one.

To check the correctness of the theoretical derivation of  $Z$  and  $\epsilon_e$ , both these quantities were measured on lines having cross sections according to Fig. 1. For comparison,  $Z$  and  $\epsilon_e$  of the standard microstrip line have been measured by the same method.

First, the permittivity  $\epsilon_r$  and the loss factor  $\tan \delta$  of the substrates "Polyplate" (pure PTFE) and "Umatex" (material comparable to copper-clad laminate G-10 according to NEMA standards), on which the samples of transmission lines were manufactured, were measured. The permittivity of the substrate was determined by measuring the resonant frequencies of a resonator formed by the substrate metallized on both bases and having open side walls [4]. The resonant frequencies were measured with errors of  $\pm 0.015$  percent, and the quality factor  $Q$  of the unloaded resonator was measured with a total error of  $\pm 15$  percent. In the frequency band 0.3–1.5 GHz, the substrates have the following parameters:

Polyplate:

$$\begin{aligned}\epsilon_r &= 2.04 \pm 0.3 \text{ percent}, \\ \tan \delta &= 0.00016 \pm 15 \text{ percent}\end{aligned}$$

Umatex:

$$\begin{aligned}\epsilon_r &= 3.97 \pm 1.5 \text{ percent}, \\ \tan \delta &= 0.036 \pm 15 \text{ percent}.\end{aligned}$$

The effective permittivity of the transmission line was determined by measuring the resonant frequency of the resonator of length  $l$  with both its ends short-circuited. Short-circuit imperfection resulted in different values of  $\epsilon_e$  calculated from resonant frequencies of the resonator having lengths equal to  $\lambda_g/2, \lambda_g, 3\lambda_g/2$ . The error of  $\epsilon_e$  caused by this imperfection did not exceed 3 percent. Therefore,

$$\epsilon_e = \left( \frac{pc}{2fl} \right)^2 \quad (20)$$

where  $p = 1, 2, 3$  if  $l = p\lambda_g/2$ .

Making use of the transformer properties of a section of lossy transmission line  $\lambda_g/4$  long (i.e.,  $\beta l = \pi/2$ , where  $\beta$  is the phase constant), its characteristic impedance can be written as

$$Z = \frac{R_i - R_t}{2} \tanh(\alpha l) \pm \sqrt{\left[ \frac{R_i - R_t}{2} \tanh(\alpha l) \right]^2 + R_i R_t} \quad (21)$$

where  $R_i$  is the measured real input resistance,  $R_t$  is the measured terminating resistance, and  $\alpha$  is its attenuation constant. The terminating resistor is realized by a parallel connection of four inductanceless resistors; therefore, its impedance can be assumed to have the real component only. The attenuation constant is determined by means of the measured quality factor  $Q$  of the line

$$\alpha = \frac{\pi f \sqrt{\epsilon_e}}{cQ}. \quad (22)$$

Energy radiation from the resonator was not observed within the accuracy of measurement; therefore, in the first approximation the quality factor in (22) represents only the losses of the line resonator assuming that the losses in both short circuits of the resonator are negligible. From (21), it is obvious that for the lossless line we get the

TABLE I  
CALCULATED AND MEASURED VALUES OF  $Z$  AND  $\epsilon_e$  OF CONVENTIONAL MICROSTRIP LINES

substrate	w/h	$\epsilon_e$		$\Delta\epsilon_e [\%]$	Z [Ω]		$\Delta Z [\%]$
		theory	measur.		theory	measur.	
Polyplate	10.5	1.89	1.94	-2.6	20.32	20.13	+0.9
	5.4	1.84	1.88	-2.1	30.44	30.46	-0.1
	2.4	1.74	1.78	-2.2	60.81	60.11	+1.2
Umatex	16.3	3.62	3.78	-4.2	10.11	10.18	-0.7
	9.8	3.50	3.65	-4.1	15.78	16.07	-1.8
	7.7	3.43	3.72	-7.8	19.32	20.07	-3.7
	2.5	3.10	3.20	-3.1	44.73	45.67	-2.1
$\epsilon_r = 2.04$							
$\epsilon_r = 3.97$							

TABLE II  
CALCULATED AND MEASURED VALUES OF  $Z$  AND  $\epsilon_e$  OF MODIFIED MICROSTRIP LINES

substrate	w/h	r/h	$\epsilon_e$		$\Delta\epsilon_e [\%]$	Z [Ω]		$\Delta Z [\%]$
			theory	measur.		theory	measur.	
Polyplate	10.5	0.9	1.84	1.92	-4.2	17.52	17.54	-0.1
	5.4	0.9	1.79	1.85	-3.2	24.44	25.50	-4.2
	2.4	0.9	1.70	1.72	-1.2	40.00	39.40	+1.5
Umatex	16.3	1.4	3.62	3.65	-0.8	9.07	9.50	-2.2
	6.8	0.2	3.27	3.18	+2.8	21.57	21.70	-0.6
	5.6	1.1	2.99	3.20	-6.6	19.76	20.60	-4.1
	2.5	1.4	2.61	2.82	-7.5	28.27	29.99	-5.7
$\epsilon_r = 2.04$								
$\epsilon_r = 3.97$								

known relation valid for a quarter-wave transformer  $Z = \sqrt{R_i R_t}$ . As in our case  $\alpha l \lesssim 0.2$ , we can write  $\tanh(\alpha l) \doteq \alpha l$ . Losses of the line being taken into consideration in (21) correct the value  $Z$  by as much as 3 percent.

To ascertain the errors in measurement of  $\epsilon_e$  and  $Z$ , both these quantities were measured on a conventional microstrip line. The values calculated according to [7] and the measured values of  $\epsilon_e$  and  $Z$  of the microstrip line are given in Table I. In calculating the relative error, the measured values have been taken as the reference.

Supposing that [7] gives the correct parameters of the microstrip line, then  $\Delta\epsilon_e$  and  $\Delta Z$  in Table I are a measure of the errors with which  $\epsilon_e$  and  $Z$  are measured. The calculated and measured values of  $\epsilon_e$  and  $Z$  of the modified microstrip line are given in Table II. The last two lines realized on Umatex substrate, parameters of which are given in Table II, did not have the upper conductor perfectly laid on the substrate, resulting in the rather high errors in  $\Delta\epsilon_e$  and  $\Delta Z$ . In spite of the measurement errors, the inaccuracy in the determination of  $\epsilon_r$ , and the simplifying assumption about the type of resistor terminating the short section of the measured line, the differences between theoretically and experimentally determined  $\epsilon_e$  and  $Z$  do not exceed  $\pm 4.2$  percent (excluding the lines produced by imperfect technology). Considering the magnitudes of  $\Delta\epsilon_e$  and  $\Delta Z$  given in Table I for the conventional line, it can be justifiably assumed that the differences between the theoretical and the actual values of  $\epsilon_e$  and  $Z$  of the modified microstrip line are less than 3 percent. Then the derived characteristic impedance and the effective permittivity sufficiently describe the parameters of the modified microstrip line suitable for transmission of high power. Within the accuracy of measurement, a frequency dependence of  $\epsilon_e$  and  $Z$  was not observed up to 1 GHz.

#### IV. APPROXIMATIONS FOR THE CHARACTERISTIC IMPEDANCE AND EFFECTIVE PERMITTIVITY

To facilitate the analysis or design of the modified microstrip line, approximations for its characteristic impedance and effective permittivity have been found. They were found by minimizing the sum of the square deviations applied successively to  $w/h$ ,  $r/h$ , and  $\epsilon_r$ . The reference data sets for  $Z$  as well as for  $\epsilon_e$  were given by the values of these quantities calculated according to (1) or (2), into which all combinations of  $\epsilon_r$ ,  $r/h$ , and  $w/h$  were substituted. Altogether, 270 values of  $Z$  and  $\epsilon_e$  were calculated setting  $\epsilon_r = 1, 2, 4, 6, 8, 10$ ;  $r/h = 0.2, 0.5, 1.0, 1.5, 2.0$ ; and  $w/h = 2, 3, 4, \dots, 10$ . The closed-form approximations for  $Z$  and  $\epsilon_e$  contain elementary functions only.

When  $\epsilon_r = 1$ , the characteristic impedance can be written in the form

$$Z_0 = \frac{71.938}{\left(\frac{w}{h}\right)^{[1.083 - 0.913(\frac{r}{h})^{0.193}]} \cdot \left(\frac{r}{h}\right)^{0.249}} - \left[ \sqrt{86.214 + 17.450\left(\frac{r}{h}\right)^2} + 2.189 \ln\left(\frac{r}{h}\right) \right] \cdot \ln\left[\left(\frac{w}{h}\right) + 0.185 \ln\left(\frac{r}{h}\right) + 0.769\right] \quad (23)$$

and  $Z_0$  is in error by less than 3 percent when  $0.2 \leq r/h \leq 2.0$  and  $2 \leq w/h \leq 10$ .

The following approximations are valid for  $1 \leq \epsilon_r \leq 10$ ,  $0.2 \leq r/h \leq 2.0$ , and  $2 \leq w/h \leq 10$ . If the individual variables are within these intervals, the deviations  $\Delta Z$  or  $\Delta \epsilon_e$  of the approximations  $Z$  or  $\epsilon_e$  from their theoretical values taken as the references do not exceed 3 percent. Thus, the characteristic impedance is

$$Z = \frac{71.6331}{\epsilon_r^{0.3285} \cdot \left(\frac{r}{h}\right)^{0.2636 - 0.0116\epsilon_r} \cdot \left(\frac{w}{h}\right)^B} - \frac{\epsilon_r D \ln E}{0.06 + 0.967\epsilon_r + 0.011\epsilon_r^{1.8}} \quad (24)$$

where

$$B = 1.3012 - 0.2204^3 \sqrt{\epsilon_r} - \left(1.4789 - 0.5573^4 \sqrt{\epsilon_r}\right) \cdot \left(\frac{r}{h}\right)^{0.185} \quad (25)$$

$$D = \sqrt{\frac{91.1274}{\epsilon_r^{0.9695}} - 0.5815\epsilon_r^{0.5761} + \left[\frac{22.45}{\epsilon_r^{1.85}} - 0.045\epsilon_r^{0.14}\right] \cdot \left(\frac{r}{h}\right)^2} + \left[\frac{1.9988}{\epsilon_r^{0.5034}} - 0.0197\epsilon_r\right] \cdot \ln\left[\left(\frac{r}{h}\right) + 0.11\right] \quad (26)$$

$$E = \left(\frac{w}{h}\right) + \sqrt{\frac{1}{1.5789 + 0.17\epsilon_r} + (0.0398 + 0.0022\epsilon_r) \left(\frac{r}{h}\right)^2} + \frac{1}{4.6634 + 0.3794\epsilon_r} - 0.22. \quad (27)$$

The effective permittivity is

$$\epsilon_e = 1 + \frac{H\left(\frac{r}{h}\right) \cdot \ln\left[\left(\frac{w}{h}\right)^M + N\right]}{0.04684 - 0.00347\epsilon_r + G\left(\frac{r}{h}\right) + K\left(\frac{r}{h}\right)^{1.957 + 0.0685\epsilon_r}} \quad (28)$$

where

$$H = 0.988 + 0.0045\epsilon_r - 0.003|\epsilon_r - 6| - \left[0.023 + 0.035\left(\frac{r}{h}\right)\right] e^{-(\epsilon_r - 5.2)^2} - 0.035e^{-(\epsilon_r - 8.9)^2} + 0.05e^{-(\epsilon_r - 6.8)^2} \quad (29)$$

$$G = \frac{e^{-0.14\epsilon_r^{1.65}}}{0.63\sqrt{\epsilon_r - 1}} + 0.038e^{-0.5(\epsilon_r - 6.4)^2} + 0.12 \quad (30)$$

$$K = \frac{e^{-0.12\epsilon_r^{1.62}}}{1.83\sqrt{\epsilon_r - 1}} + 0.031e^{-0.5(\epsilon_r - 6.98)^2} + 0.123 \quad (31)$$

$$M = 0.38 + 0.05 \ln(\epsilon_r + 1.1) - 0.486\left(\frac{r}{h}\right)^{(0.61 - 0.01\epsilon_r)} + (0.2696 + 0.023\epsilon_r) \cdot \left(\frac{r}{h}\right)^{1.31} + (0.0627 - 0.0084\epsilon_r + 0.001\epsilon_r^2) e^{-3S^2} \quad (32)$$

$$S = \left(\frac{r}{h}\right) + 0.37 + 0.037\epsilon_r - 0.91 \ln(\epsilon_r + 3.0) \quad (33)$$

$$N = 0.682 - 0.065\epsilon_r + 0.0008\epsilon_r^{2.395} + (0.266 + 0.054\epsilon_r) \left(\frac{r}{h}\right)^{1.2} + \frac{0.738 - 0.065\epsilon_r + 0.0008\epsilon_r^{2.385}}{\left(\frac{r}{h}\right)^{[0.4 \ln(\epsilon_r + 1.0) - 0.217 + 0.0155\epsilon_r]}}. \quad (34)$$

Values of  $Z_0$  calculated according to (24) differ from those obtained from (23), but in both cases they deviate from the values of  $Z_0$  calculated according to (1) by less than 3 percent. The characteristic impedance can also be calculated from the formula

$$Z = \frac{Z_0}{\sqrt{\epsilon_e}} \quad (35)$$

in which  $Z_0$  is given by expression (23) and  $\epsilon_e$  by expression (28). In this case, the deviation  $\Delta Z$  of the characteristic impedance from the reference data is less than 3 percent. If expression (24) is substituted for  $Z_0$  in (35) with  $\epsilon_r = 1$ , the deviation  $\Delta Z$  will exceed 3 percent for some combinations of  $\epsilon_r$ ,  $r/h$ , and  $w/h$ . On the other hand, it is not recommended to calculate the effective permittivity according to (2) using expressions (23) and (24); nor is it recommended that expression (24) be used for evaluating both  $Z$  ( $\epsilon_r \neq 1$ ) and  $Z_0$  ( $\epsilon_r = 1$ ), since this would lead to an error  $\Delta\epsilon_e > 3$  percent. Expressions (23), (24), and (28) describe the dependence of  $Z_0$ ,  $Z$ , and  $\epsilon_e$  on the particular

variables  $\epsilon_r$ ,  $r/h$ , and  $w/h$  within their limits quoted above. The validity of the above-mentioned expressions outside the specified limits has not been investigated.

Having the effective permittivity  $\epsilon_e$  obtained from (28), one can calculate the portion of the attenuation constant of the modified microstrip line related to the losses in the dielectric substrate [4] using the formula

$$\alpha_d = \pi \frac{f}{c} \frac{\epsilon_r}{\sqrt{\epsilon_e}} \frac{\partial \epsilon_e}{\partial \epsilon_r} \tan \delta. \quad (36)$$

The phase velocity of the quasi-TEM wave propagating on the modified microstrip line is  $v = c/\sqrt{\epsilon_e}$  and its wavelength is  $\lambda_g = c/f\sqrt{\epsilon_e}$ .

## V. CONCLUSIONS

Calculation of the characteristic impedance and effective permittivity in the quasi-TEM approximation of the modified microstrip line characterized by the edges of its upper strip conductor turned up from the substrate is presented. This modification of the line cross section results in higher transmitted power, especially if it is manufactured on a low-cost laminated plastic substrate. The theoretical and the measured values of the characteristic impedance and effective permittivity are compared. From their satisfactory agreement, one can conclude that the derived results are appropriate solutions to the problem for the case of "wide" modified microstrip lines. It is believed that this is the first description of the quantities quoted above for this type of line.

To facilitate analysis and design of the modified microstrip lines, approximations in closed form to the characteristic impedance and effective permittivity containing elementary functions have been found. When  $\epsilon_r$ ,  $r/h$ , and  $w/h$  are within the limits of validity of the approximations, the approximations deviate from the theoretically calculated set of data by less than 3 percent. They shorten the computation time of all line parameters considerably, and permit analytic determination of the attenuation constant corresponding to dielectric losses. The modified microstrip line has lower dielectric losses than conventional microstrip line with the same characteristic impedance. Its geometry permits simple mounting of transistors in the stripline version.

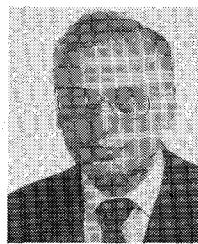
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